Applying Mathematical Epidemic Modeling to Discover Commercially Beneficial Outbreak Control Methods

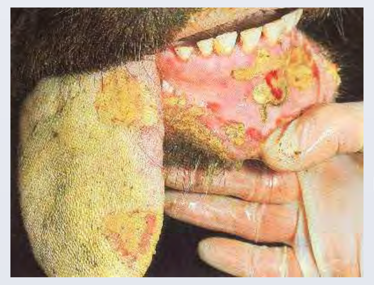
Michael W Ciesa
Advisor: Dr. Muhammad Usman
Department of Mathematics, University of Dayton

Abstract

This is a mathematical analysis and computational study of the 2001 foot and mouth epidemic in the UK. This model includes an application of the SIR model, developed by W. O. Kermack and A. G. McKendrick, with three additional factors: vaccination, culling and incubation period. The incubation period refers to the latent population which represents the population of individuals infected with the disease but do not show symptoms, they are still nonetheless part of the infected population. It is the goal of this analysis to more accurately determine the rate at which vaccination and culling should be applied and which methods will result in greater commercial value for the livestock populations.

Back Ground

Foot-and-mouth disease (FMD) is a highly infectious disease caused by an aphthovirus that affects cloven-hoofed animals such as pigs, cattle and sheep [2]. It can have disastrous effects on a countries food supply and economic stability. The foot-and-mouth outbreak of 2001 in the UK resulted in nearly 4 million sheep and cattle being culled and killed in the eventually successful attempt to stop the spread of the disease [1]. Infected animals cannot be sold for food, milk or wool and must be separated from the main population, as their presence can cause the spread of the infection throughout the livestock population. While there is evidence that vaccinated animals are safe for consumption [3]; once vaccinated the animal becomes unsuitable for export and thus loses a portion of their economic value [1]. Culling is an outbreak control technique that employs the separation of infected individuals from the susceptible and healthy population; the infected animals are often slaughtered and disposed of far from the herd. Culling and vaccination can be employed in conjunction to successfully battle the spread of the disease during an outbreak. Mathematical epidemic modeling can provide government officials and farmers with beneficial information on how to effectively combat the spread of the disease during an outbreak.



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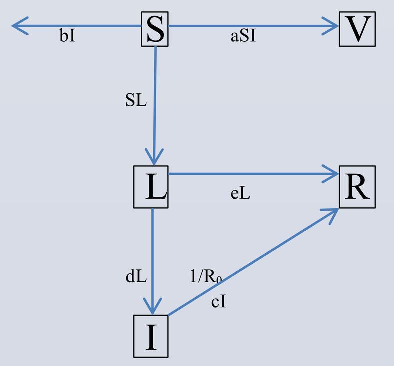
SLIR Model

The SLIR model (or SEIR) is derived from the basic SIR epidemic model with an additional compartment, L which represents the latent or exposed population. The susceptible population (S) represents the population of livestock that may catch the disease; the infective population (I) represents the livestock population that is infected with the disease; and the removed population (R) represents the livestock population that either died or recovered from the disease. This model was non-dimensionalized and modified to include culling and vaccination control techniques. This model is a qualitative analysis of the effectiveness of these techniques in controlling the spread of foot-and-mouth disease during an outbreak.

$$\frac{dS}{dt} = -SL - bI - aSI$$
$$\frac{dL}{dt} = SL - dL - eL$$
$$\frac{dI}{dt} = dL - \frac{1}{R_0}I - cI$$
$$\frac{dR}{dt} = eL + \frac{1}{R_0}I + cI$$
$$\frac{dV}{dt} = aSI$$

a = vaccination rate
b = rate of culling susceptible
c = rate of culling infectives
d = incubation period
e = rate of culling latents

Compartmental Flow Diagram



Assumptions of Model

- There is a rate of vaccination of susceptibles proportional to both the number of susceptibles and number of infectives, as the vaccination rate will depend on the severity of the epidemic and the ability to locate susceptibles.
- There is a constant rate of removal of animals by culling in addition to the natural removal rate.
- Model sets birth rate conditions so that an epidemic occurs ($R_0 = 3$)
- The incubation/latency period is an average, whereas the actual incubation/latency period may show greater variability depending upon the individual.
- The rate of culling of latents must be less than the rate of culling of infectives, due to the complexity of the identification process of latent individuals.

Results

Without any control policies the spread of foot-and-mouth disease will have a catastrophic toll on the livestock population as shown in figure 1. The following figures are simulations, ran through a Matlab 7 program using an ode23 solver. Many trials were conducted and the most successful trials are shown. Figures 2 and 3 show which control policy, vaccination or culling, is most effective during an outbreak; figures 4 and 5 show the rate at which culling and vaccination must occur for there to be more healthy livestock than infected livestock; and figures 6 and 7 show the effects that a high vaccination rate has on the commercial values of the livestock populations. For convenience the variable meanings are relisted below and the numerical values of those variables are listed below each graph.

a = vaccination rate
b = rate of culling susceptible
c = rate of culling infectives
d = incubation period
e = rate of culling latents

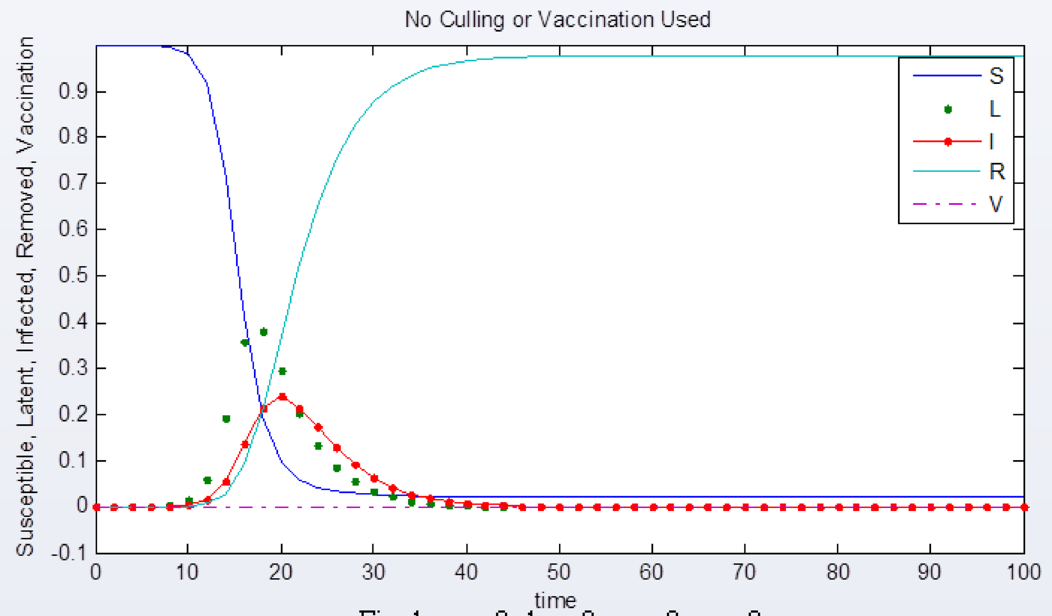


Fig 1: a = 0, b = 0, c = 0, e = 0

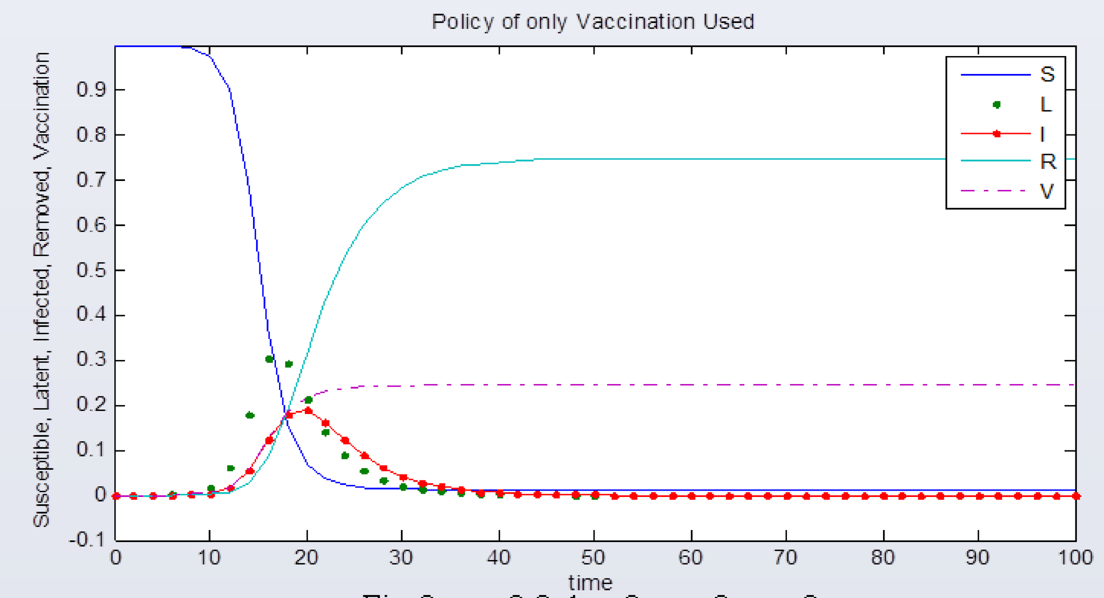


Fig 2: a = 0.8, b = 0, c = 0, e = 0

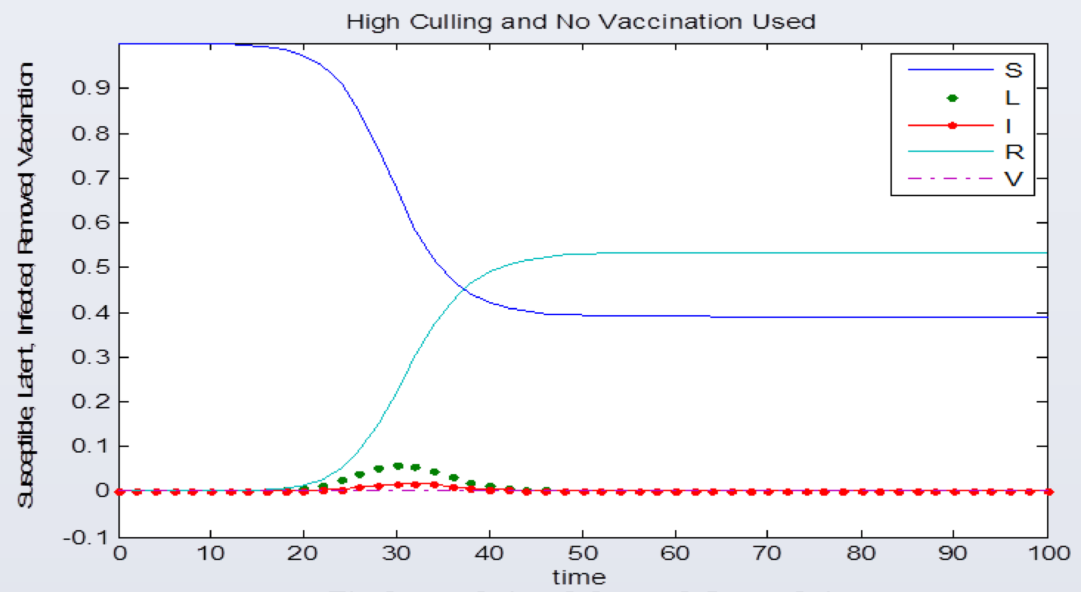


Fig 3: a = 0, b = 0.3, c = 0.5, e = 0.4

The simulations here show that a policy of vaccination during an outbreak is not a very effective control method, while a policy of culling during the outbreak appears to be a much more effective method of control.

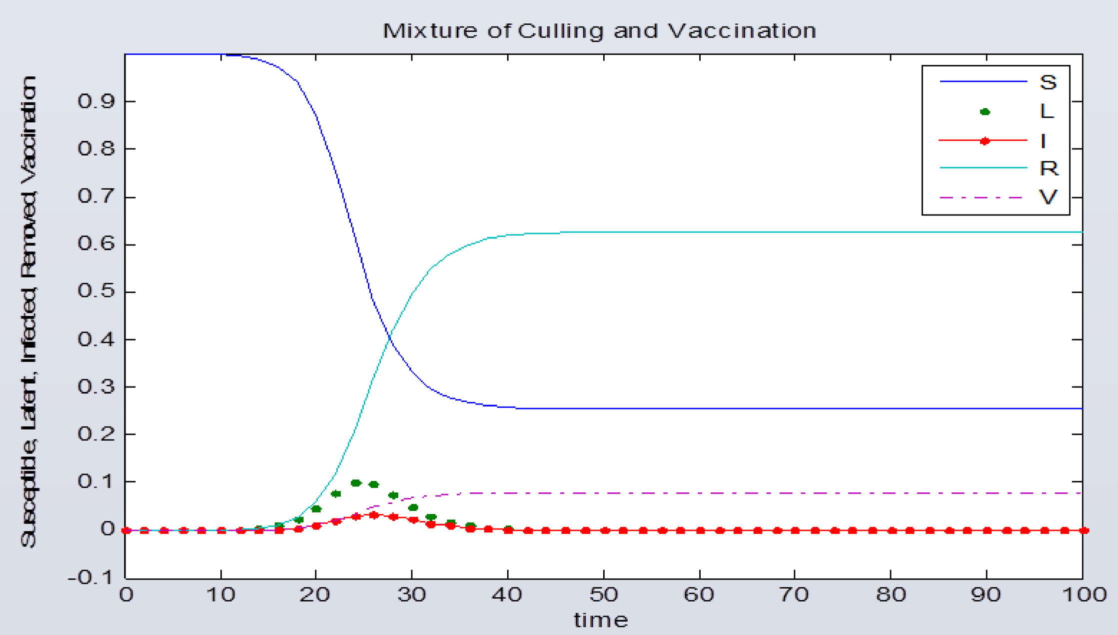


Fig 4: a = 0.4, b = 0.1, c = 0.4, e = 0.3

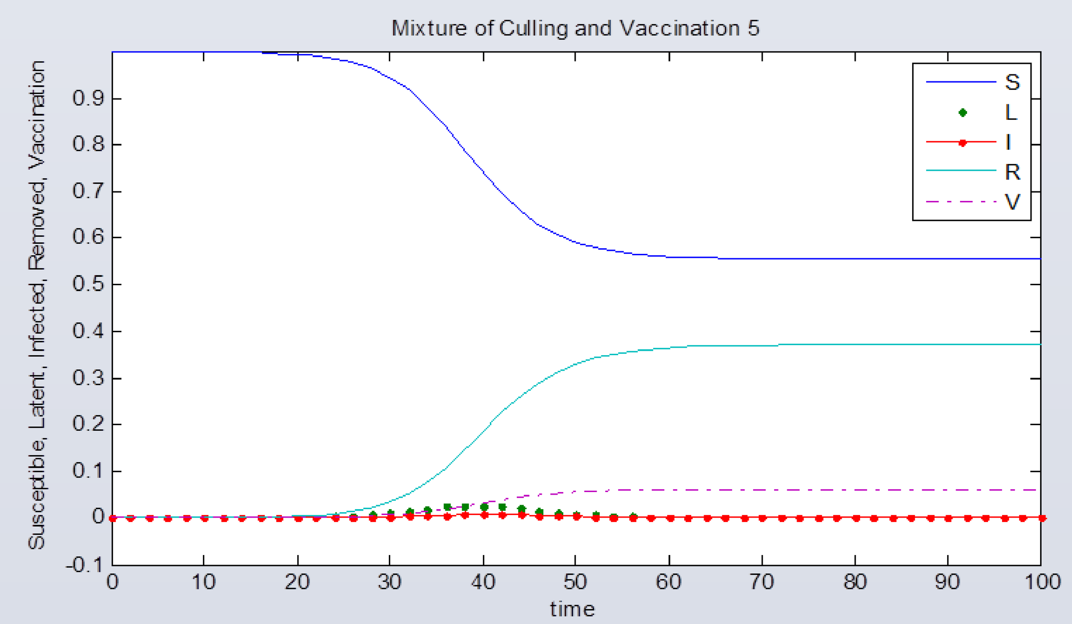


Fig 5: a = 0.6, b = 0.1, c = 0.6, e = 0.5

An equal mixture of culling and vaccination during an outbreak appears to be the most effective policy. Although, the rate of culling and vaccination must be high in order to maintain a favorable susceptible to removed population ratio.

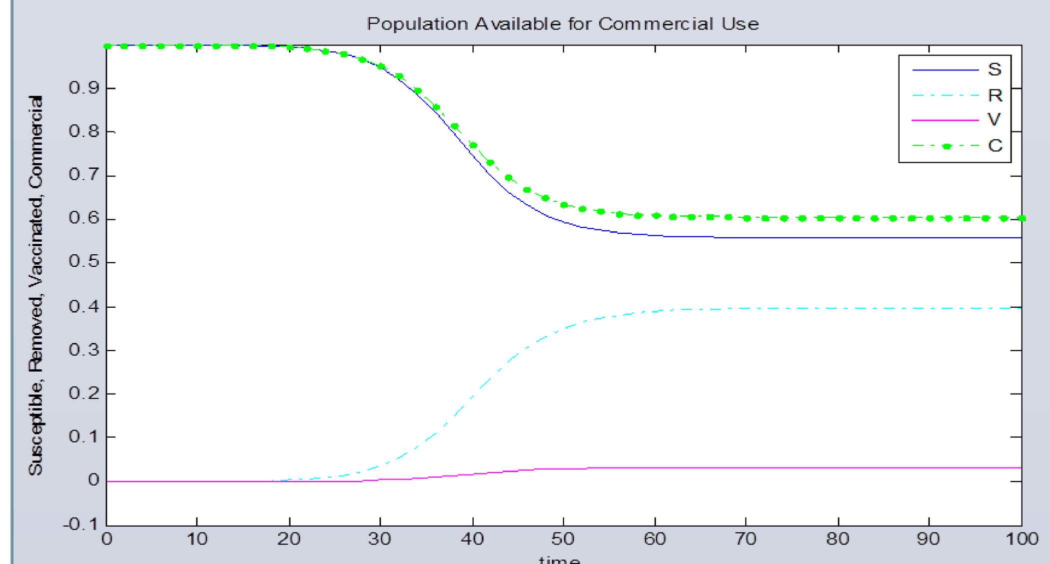


Fig 6: a = 0.3, b = 0.1, c = 0.6, e = 0.5

A high rate of vaccination increases the amount of livestock available for commercial use by approximately 0.03

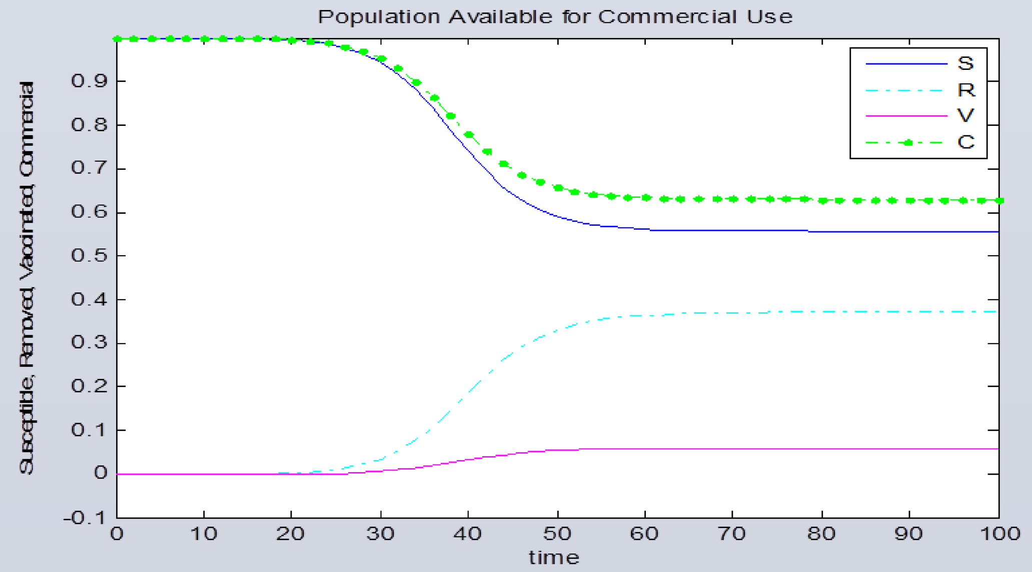


Fig 7: a = 0.6, b = 0.1, c = 0.6, e = 0.5

Conclusion

When no control policies are employed the catastrophic results are obvious. A mixture of vaccination and culling adds significantly positive results towards the containment of the disease. While culling is a much more effective method of control, vaccination can be a valuable preventative policy. Any animal that remains disease free or becomes vaccinated has some commercial value and therefore a policy of vaccination during an outbreak, while not contributing greatly to the control of the disease, can still contribute positively towards the economic value of the livestock population. The problem vaccination can pose is in the cost it takes to vaccinate the livestock. While the manufacturing cost of the vaccine is fairly cheap, about 60 cents [4], I cannot speak to what a country's government or a private company would sell the vaccine for. Additionally, the cost of employing people to apply the vaccine has to be considered. As the simulations indicate, employing a high level of vaccination in addition to a high level of culling results in an approximate 3% increase of livestock available for commercial use. While vaccinated animals cannot be exported they can still be used locally and therefore do not lose all of their economic value. When deciding whether to employ a high level of vaccination along with culling, the profits gained from the commercial use of the livestock must be greater than the cost of vaccination. When making this decision market values of the livestock must be researched in order to determine which livestock species' market values justify extra vaccination. It is important to remember that this is a simple qualitative model and in reality there are numerous factors, such as spatial effects, that could skew these trends. It is my opinion that large profit livestock, such as steers and dairy cows, will create large enough commercial profits to justify the additional cost to be vaccinated at a high rate.

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References and Acknowledgments

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